

The Northern Immigration Policy in a North-South Economy Model

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Wooton (1985) considered the immigration into Findlay (1980)'s North-South model and examined how the north's relaxation of immigration policy influences income in both regions. Based on Wooton (1985), this paper has performed the same analysis assuming there to be a complete capital mobility between the north and the south. The major findings are as follows: In the long-run, the relaxation of the north's immigration policy does not affect the per capita income of both the northern labour and immigrant workers. When the north practises a discriminative redistribution policy against the immigrant workers, the per capita income in the north will increase because redistribution of income from the immigrant workers to the northern labour as a result of the policy relaxation is taking place.

1. INTRODUCTION

The issue of how the influx of immigrants from the southern economies to the north has affected the economy in a north-south model has always been discussed. As the globalisation of the world economy accelerates, this issue is back in the spotlight.

In a Hecksher-Ohlin model, which is commonly used when international trade is coming into play, free trade is said to equalise factor prices and hence there is no incentive for factors to move from one country to another. In such a model, it is assumed that all countries are identical except in their two factor endowments—capital and labour.

Contrary to such supposition, Findlay (1980) features a model that includes the asymmetry between the north and the south. There are two assumptions that illustrate this asymmetrical relationship between the two regions. For one, it is assumed that complete specialisation holds such that the north produces an industrial

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good that features both as consumption and capital inputs while the south imports capital good from the north and produces a primary good, which also is a consumption good, using the imported capital goods. The second assumption has got something to do with the elasticity of the labour supply. The northern economy is assumed to be a fully employed economy in which the labour supply is inelastic and all labour is used in the production process. Therefore, in the long-run the growth rate is determined by the growth of labour supply, which grows at a constant rate. On the other hand, the south is a labour surplus economy. The modern sector of the economy that produces primary goods can obtain unlimited supply of labour from the traditional sector that produces non-tradable goods at a fixed amount of subsistence wage rate.

Wooton (1985) uses this Findlay (1980) model and examines how the relaxation of immigration policy influences the north-south economy. In Wooton (1985), it is assumed that the northern labour supply consists of immigrant workers at a certain ratio and this ratio is treated as the policy parameter. An assumption like this is appropriate for the northern governments' management of immigrants, such as that adopted by the West Germany or Switzerland after the World War II. Immigrants reside for a period of time in the north, receive income and remit that income to the south. The northern labour receives both wage and profit income. Wooton (1985) reviews how the north's relaxation of immigration policy influences the per capita income of both the northern and the southern labour and the potential profit of the policy. The major findings of Wooton (1985) are as follows.

In the short-run when the capital stock is fixed, relaxation of immigration policy will ease the pressure on the north's labour supply constraint and increases the northern labour's per capita income. Although the wage rate falls the rise of profit rate surpasses the fall thereon. Since the north's per capita income increases, the south's terms of trade will also improve provided that the total income of the immigrants does not decrease significantly.

In the long-run, the north's immigration policy will not affect the terms of trade because the terms of trade will be adjusted to equalise the growth rate in the two regions. The increase of profit rate in the north will raise its capital-labour ratio in the steady state, hence the north's wage rate partially recovers from its temporary fall. The south's employment will increase in this case provided the total income of the immigrants does not decrease significantly.¹

From what we have seen above, we can conclude that the north enjoys the benefits of labour mobility between the two regions. The south, even when the immigrant labourers reduce the remittance of income, will enjoy benefit gained from a constrained labour mobility regardless of the increase in profit reaped from the rise of import demand from the north.

¹ For a more detailed discussion on foreign trade and investment issues, see Khan (1984).

On top of that, the tax policy of both the north and the south is also discussed. Taking it into consideration, the optimum policy for the north would be to completely free the labour mobility and administer a non-discriminative redistribution policy. A tax levied on the emigrating workers by the south government will in turn reduce the income of the north and also its import demand. As a result, the south's employment could potentially be hurt.

Above is the main argument of Wooton (1985). Freedom in capital mobility has become an issue to be reckoned with in today's globalisation of economy. Burgstaller and Saavedra-Rivano (1984) discuss the capital mobility which includes the structural differences between the north and the south. With an assumption that the goods have high malleability, the said paper introduces capital mobility between the two regions into Findlay (1980) model. This paper examines how the capital mobility between the two regions influences the per capita income thereon but does not discuss the immigration policy.

Based on Wooton (1985) and Burgstaller and Saavedra-Rivano (1984) this paper attempts a parallel examination of how the relaxation of immigration policy in the north and the tax policy of both governments affect the per capita income in both the regions, given there is a complete capital mobility between the north and the south.

The paper is organised into three sections. Section 2 explains the Wooton (1985) model. In Section 3 the analysis is performed adopting the capital mobility into the model and Section 4 concludes the analysis.

2. THE WOOTON MODEL

This section explains the Wooton (1985) model. Subsection 2.1 states the basic structure of the model. Section 2.2 elaborates the relaxation of immigration policy, and Section 2.3 the tax policy.

2.1. Basic Structure of the Model

The northern economy. The north uses capital and labour, K and L respectively, to produce industrial goods. L consists of the northern labourers as well as the immigrant workers. Therefore, the production function of the north is given by

$$Y=F(K, L) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

Suppose the northern production function is a constant-returns to scale production function, furthermore let f denote the per capita production of the north and k denote the capital-labour ratio, we can derive from (1)

$$f=f(k) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

It is assumed that the production function satisfies the Inada condition.

Since the real wage rate, w , of the north equals its marginal productivity of labour,

$$w = f'(k) - f''(k)k \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (3)$$

And since the interest rate, r , equals the marginal productivity of capital,

$$r = f''(k) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (4)$$

Suppose that the labour force of the north, L_N , grows at n , an exogenously determined growth rate. Hence, suppose the initial value of the north's labour force is \tilde{L}_N ,

$$L_N = \tilde{L}_N \exp(nt) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (5)$$

The north temporarily absorbs L_G^* of immigrant workers from the south and all of it is employed in the north's production sector. Suppose the north only absorbs the immigrant worker up to a certain fraction to the local labour supply, τ ,

$$L_G^* = \tau L_N, \quad \tau > 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6)$$

Both L_G^* and L_N grow at the same rate.

Denote k_N as the capital per northern labour, the per capita income of the north, y , is given by

$$y = w + rk_N \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (7)$$

The northern labour enjoys both wage and profit income. Since $L = L_N + L_G^*$, k is related to k_N in such a way that

$$k = \frac{K}{L_N + L_G^*} = \frac{K}{(1 + \tau)L_N} = \frac{k_N}{1 + \tau} \quad \dots \quad \dots \quad \dots \quad \dots \quad (8)$$

Because the immigrant workers do not own capital, their per capita income, y_G^* , is given by

$$y_G^* = w \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

The immigrant workers send a fraction of their income, q , back to their home country. Denote Q as the total remittance of money by immigrant workers to the south,

$$Q = qy_G^* L_G^* \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (10)$$

The southern economy. The modern sector in the south uses capital and labour, K^* and L^* to produce a primary good. Hence the production function is given as

$$Y^* = F^*(K^*, L^*) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (11)$$

Suppose this is a constant-returns-to-scale production function, denote f^* as per capita production and k^* as capital-labour ratio, it can be rewritten as

$$f^* = f^*(k^*) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (12)$$

The south's production is consumed domestically and exchanged for the north's consumption and capital goods.

There is a traditional sector in the south called the hinterland that produces only non-tradable goods. The hinterland absorbs all labours except those who work in the production of traded goods. Therefore, the modern sector could pull as much labour supply at a fixed wage rate from such hinterland.

Since the real wage rate w^* in the south equals its marginal productivity of labour,

$$\bar{w}^* = f^*(k^*) - f^{*'}(k^*)k^* \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (13)$$

The south's profit rate, r^* , equals its marginal productivity of capital. Denote p as its terms of trade,

$$r^* = pf^*(k^*) \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (14)$$

The per capita income, y^* , of the south's modern sector consists of the per capita factor income $f^*(k^*)$ and the remittance from the emigrant to the north. Hence, denote λ as the employment ratio between the two regions, we get

$$y^* = f^*(k^*) + \frac{qY_G^\tau}{\lambda p} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (15)$$

Suppose a fraction of profit income, s^* , is spent on the investment goods demand, the south's per capita expense on consumption goods, e^* , is given by

$$e^* = f^*(k^*) - s^* f^{*'}(k^*)k^* \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (16)$$

Balance of payments equilibrium. Denote m as the demand function of consumption imports per northern labourer. Suppose a certain fraction of s of the north's income is saved, the per capita consumption expense in the north is then $(1-s)y$. m is a decreasing function of the south's terms of trade, p , and an increasing function of $(1-s)y$. Here we assume the elasticity of consumption expenses takes a

value of one. Next, denote \tilde{m} as the demand function of consumption imports per immigrant worker from the south. \tilde{m} is a decreasing function of p and an increasing function of the disposable income per immigrant worker, $(1-q)y_G^*$. Denote μ as the import demand for consumption per southern labourer, and assume that μ is a decreasing function of $\frac{1}{p}$ and an increasing function of e^* . Taking the remittance from the immigrant workers back to the south, the balance of payments equilibrium is given by

$$p\{m[p, (1-s)y] + \tilde{m}[p, (1-q)y_G]\tau\} = \left\{s^* r^* k^* + \mu\left[\frac{1}{p}, e^*\right]\right\}\lambda - qy_G^* \tau \quad \dots \quad (17)$$

Capital accumulation. Next we will look at the capital accumulation. In the north, a certain fraction s of the income is saved and invested thereon. Since the northern labour growth rate is n , we can express the dynamic equation of k_N as

$$\dot{k}_N = sy - nk_N \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (18)$$

In the south, although a certain fraction s^* out of the profits is diverted to capital accumulation, the south's employment growth rate equals its capital accumulation rate because from (13) we know that k^* is a fixed value. Thus the dynamic equation pertaining to the north-south employment ratio, λ , is given by

$$\dot{\lambda} = (s^* r^* - n)\lambda \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (19)$$

Determinant relations of the model. With a total of 17 equations namely (2)~(10) and (12)~(19) together with a total of 17 endogenous variables, $f(k)$, k , w , r , L_N , L_G^* , y , k_N , y_G^* , Q , $f^*(k^*)$, k^* , r^* , p , λ , y^* and e^* , this model is closed².

The major determinant relations are as follows. From Equations (5), (18) and (19), suppose that L_N , k_N and λ each takes a certain value. L_G^* will then be determined in (6) and k in (8). Hence, from Equations (2), (3), (4), (7) and (9) $f(k)$, w , r , y and y_G^* are determined, respectively. Having the above-mentioned results, Q is determined in Equation (10). k^* is then determined in Equation (13), and Equation (12) solved $f^*(k^*)$. Lastly, p is determined in Equation (17) because e^* is a function of p as we know from Equations (15) and (16).

²Please refer to Appendix 1 for stability conditions.

2.2. The North's Relaxation of Immigration Policy

In this section, we will see how the relaxation of the north's immigration policy influences the per capita income in both regions in two scenarios—in the short-run where both k_N and λ are fixed and in the long run where both change.³

Short-run equilibrium analysis. First of all, we will see how the per capita income, y_G^* of the immigrants is affected. From Equations (3), (8) and (9) we can derive

$$\frac{dy_G^*}{d\tau} = \frac{k^2 f''(k)}{1 + \tau} < 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (20)$$

This shows that the increase of immigrants will reduce the north's real wage rate.

Next, we will see how the income per northern labourer, y , is affected. From Equations (3), (4) and (8) we know that

$$\frac{dy}{d\tau} = \frac{-\tau k^2 f''(k)}{1 + \tau} > 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (21)$$

This tells us that the decrease in wage income is offset by the increase in profit income.

Denote η as the price elasticity of the imports demand of the northern labour, $\tilde{\eta}$ as the price elasticity of the imports demand of the immigrants, by virtue of Equation (17) we can derive the following.

$$\frac{dp}{d\tau} = \frac{pm_2(1-s)\frac{dy}{d\tau} + \left\{ pm_2(1-q) + q\left(1 - \frac{\mu_2}{p}\right) \right\} \left(\tau \frac{dy_G^*}{d\tau} + y_G^* \right)}{\frac{d\{s^* r^* k^* + \lambda \mu\}}{dp} - \left\{ m[p, (1-s)y](1-\eta) + \tau m[p, (1-q)y_G^*](1-\tilde{\eta}) \right\}} \dots \quad (22)$$

By virtue of the stability conditions of the balance of payments equilibrium the denominator of Equation (22) takes a positive value. Although the first term of the numerator takes a positive sign, both the sign of the second term of the numerator and $\frac{dp}{d\tau}$ cannot be determined because there is no way to tell if the total income of the immigrants has increased or otherwise.⁴ Hence, from Equation (15) it is not possible to tell how the per capita income in the south, y_G^* , is affected.

³ Throughout this paper it is assumed that goods have high malleability that it is possible for capital to move spontaneously. This sort of assumption poses a problem in a short-run analysis. However, since the purpose of this section is to compare the Wooton (1985) results, we will come back to this problem in future studies.

⁴ Furthermore, $1 - \frac{\mu_2}{p} > 0$. Please refer Appendix 2.

Table 1 sums up the short-run results of Wooton (1985).⁵

Table 1

Wooton(1985) (Short-run)

	y	y_G^*	py^*	p
τ	+	-	\pm	\pm

Long-run equilibrium analysis. Long-run equilibrium is defined as a state in which the capital accumulation rate in the north equals that in the south. From Equations (3), (4), (7), (8) and (18) we can derive the followings.

$$\frac{dk_N}{d\tau} = \frac{\frac{-\tau sk^2 f''(k)}{1+\tau}}{n-s\left\{f'(k) + \frac{\tau}{1+\tau}kf''(k)\right\}} > 0 \quad \dots \quad \dots \quad \dots \quad (23)$$

$$\frac{dy}{d\tau} = \frac{n}{s} \frac{dk_N}{d\tau} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (24)$$

$$\frac{dk_N}{d\tau} = \frac{\frac{\{sf'(k) - n\}k}{1+\tau}}{n-s\left\{f'(k) + \frac{\tau}{1+\tau}kf''(k)\right\}} > 0 \quad \dots \quad \dots \quad \dots \quad (25)$$

$$\frac{dw}{d\tau} = -kf''(k) \frac{dk}{d\tau} < 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (26)$$

Equations (23) and (24) illustrate that the increase of immigrants will push the north's capital accumulation and thus increase its per capita income. On the other hand Equations (25) and (26) show that the increase of immigrants reduces the real wage rate. From Equation (17) we can derive the north-south employment ratio as

$$\frac{d\lambda}{d\tau} = \frac{pm_2(1-s)\frac{dy}{d\tau} + \left\{pm_2(1-q) + q\left(1 - \frac{\mu_2}{p}\right)\right\}\left\{y_G^* + \tau\frac{dy_G^*}{d\tau}\right\}}{\mu\left[\frac{1}{p}, e^*\right]\left\{1 - \frac{\mu_2 r \tau y_G^*}{\mu p \lambda}\right\} + nk} \quad \dots \quad (27)$$

⁵Wooton (1985) uses λy^* , the southern income per northern labour to express the southern welfare. However, $\lambda^{\alpha} y^{*\beta}$ is more commonly used and it is thought that the southern welfare depends on the sizes of α and β . Therefore this paper does not use λy^* to represent the southern welfare.

By virtue of stability condition the denominator takes a positive value. Although the first term of the numerator to the right takes a positive sign, it is not possible to determine the sign of the second term because it is not possible to determine whether the income of the immigrants, τy_G^* , will increase or otherwise.

Hence, from (15) the effect of y^* is also undetermined.

The terms of trade of the south do not change because they are determined by Equations (13) and (19).

From the above results, we can conclude that although the north favours a free labour mobility, the south might favour a constrained labour mobility because of the suppressing effect of the remittances by the immigrant workers.

Table 2 sums up the long-run results of Wooton (1985).

Table 2

Wooton(1985) (Long-run)

	y_N	y_G^*	py^*	p	λ
τ	+	-	\pm	0	\pm

2.3. Tax Policy

This section examines about the tax policy of both governments.

The northern tax policy. Suppose that the northern government levies a proportional tax on labour income and redistributes it among the northern workers. This basically says that the immigrant workers are discriminated in terms of wage. Assuming a t percent of the wage income is taxed, the per capita income of the immigrant workers will then be

$$\hat{y}_G^* = (1-t)w \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (28)$$

The per capita income of the northern workers is given by the following because the tax is redistributed among the northern workers.

$$\hat{y} = \frac{wL_N + twL_G^* + rK}{L_N} = (1+t\tau)w + rk_N \quad \dots \quad \dots \quad \dots \quad (29)$$

We will discuss the long term effects of the north's relaxation of immigration policy on its per capita income. From Equations (3), (4), (8), (18) and (29) we can derive

$$\frac{dk_N}{dt} = \frac{s \left\{ tw - \frac{\tau}{1+\tau} (1-t)k^2 f''(k) \right\}}{n - s \left\{ f'(k) + \frac{\tau}{1+\tau} (1-t)kf''(k) \right\}} > 0 \quad \dots \quad \dots \quad \dots \quad (30)$$

Furthermore, from (8)

$$\frac{d\hat{y}}{d\tau} = \frac{n}{s} \frac{dk_N}{d\tau} > 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (31)$$

Hence the per capita income in the north increases as a result of the increase in immigrants. So it is fair to say that the optimal policy for the north will be to completely free the labour mobility in two regions even when the tax issue is taken into consideration. When labour mobility is completely free, labour movement from the south to the north stops at the point where the real wage rates in both the regions are equal. Hence

$$\hat{y}_G^* = pw^* \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (32)$$

Next we shall see what happens when tax policy is introduced in a scenario illustrated by Equation (32)—a scenario where labour mobility is completely free. τ is endogenised so that (32) will hold. From Equations (3), (13), (19), (28) and (32) we can derive

$$\frac{dk}{dt} = \frac{-w}{(1-t)kf''(k)} > 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (33)$$

It shows that the increase of t reduces the per capita income of the immigrants, \hat{y}_G , meanwhile a movement of the immigrants from the north to the south occurred. Hence by using Equations (7), (8), (18) and (33) we can derive the following equations and show that an increase of tax reduces the north's per capita income.

$$\frac{dk_N}{dt} = -\frac{t}{1-t} (1+\tau)^2 \frac{dk}{dt} < 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad (34)$$

$$\frac{d\hat{y}}{dt} = \frac{n}{s} \frac{dk_N}{dt} < 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (35)$$

This is due to the fact that tax hike reduces profit rate and in turn reduces capital accumulation.

Based on the above results, we can conclude that the optimum policy for the north is to completely free labour movement and to implement a non-discriminative redistribution policy.

The southern tax policy. The southern government taxes the emigrant workers and redistributes it to the southern workers. This can be illustrated by the increase in q . Using Equation (17)

$$\frac{d\lambda}{dq} = \tau y_G^* \frac{1 - (p\tilde{m}_2 + \frac{\mu_2}{p})}{s^* r^* k^* + \mu - \mu_2 y_G^* \frac{q\tau}{p\lambda}} \quad \dots \quad \dots \quad \dots \quad \dots \quad (36)$$

The denominator takes a positive value by virtue of the stability condition. However, the sign of $\frac{d\lambda}{d\tau}$ is unable to be determined because there is no saying what sign will the numerator take.

In a scenario where labour movement is completely free between the two regions, Equation (32) illustrates that it is possible that $(1-q)\hat{y}_G^* = pw^*$. This is different from Equation (32) because the compulsory taxation, q , exists. Therefore, both y and τ change depending on the changes of q . We can derive the following using Equation (17)

$$\frac{d\lambda}{dq} = \frac{pm_2(1-s)\frac{dy}{dq} + (p\tilde{m} + qy_G^*)\frac{d\tau}{dq} + (1 - \frac{\mu_2}{p})\frac{\tau y_G^*}{1-q}}{s^* r^* k^* + \mu_2 y_G^* \frac{q\tau}{p\lambda}} \dots \dots (37)$$

The first and second terms of the numerator takes a minus sign as a result of the north's rise of wage rate due to the increase of q . The third term takes a positive sign because it shows the relaxation of the balance of payments constraint due to the increase of remittance back to the south. Hence the sign of Equation (37) cannot be determined. From these results we know that the employment in the south decreases should the tax policy in the south significantly reduces the income in the north.

3. IMMIGRATION POLICY WITH FREE CAPITAL MOVEMENT

This section performs the same analysis with a free capital movement. The northern economy remains basically the same as in the last section.

3.1. Basic Structure of the Model

Capital movement. Taking a little hint from Burgstaller and Saavedra-Rivano (1984), suppose that capital movement happens arbitrary so that the profit rate equals in both regions. We get

$$r^* = r \dots \dots \dots (38)$$

Denote K_S^* , K_N , K and K^* as the south's capital stock, the north's capital stock, capital stock used in the production of industrial goods and capital stock used in the production of primary goods, respectively, the capital balance equation is

$$K_S^* + K_N = K + K^* \dots \dots \dots (39)$$

Divide both sides of (39) by L_N we can derive the following.⁶

$$k_S^* + k_N = (1 + \tau)k + k^* \lambda \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (40)$$

Where, $k_S^* = \frac{K_S^*}{L_N}$, $k_N = \frac{K_N}{L_N}$, $k = \frac{K}{L_N + L_G^*}$ and $\lambda = \frac{L^*}{L_N}$.

The southern economy. The southern income consists of wage income, profit income and remittance from the emigrant workers. Hence its per capita income is given by

$$y^* = f^*(k^*) - f^{*'}(k^*)k^* + f^{*'}(k^*)\frac{k_S^*}{\lambda} + \frac{qy_G^*\tau}{\lambda p} \quad \dots \quad \dots \quad (41)$$

The first and second term to the right is the real wage rate in the south, the third is the south's profit income and the last is the remittance from the workers who have emigrated to the north. Therefore, the per capita consumption expenditure in the south is

$$e^* = y^* - s^* f^{*'}(k^*)\frac{K_S^*}{L^*} = y^* - s^* f^{*'}\frac{k_S^*}{\lambda} \quad \dots \quad \dots \quad \dots \quad (42)$$

The second term to the right is the demand for investment goods in the south, s^* represents the fraction of profit income spent on the demand for investment goods.

Balance-of-payments equilibrium. Adopting the profit gained in the south by the north, balance of payments equilibrium is given by:

$$p\left\{m[p, (1-s)y] + \tilde{m}[p, (1-q)y_G^*]\tau\right\} = \left\{s^* pf^{*'}(k^*)k_S^* + \lambda\mu\left[\frac{1}{p}, e^*\right]\right\} \\ - qy_G^*\tau + pf^{*'}(k^*)\lambda - k_S^* \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (43)$$

The third term on the right hand side is the remittance to the south by the immigrants, the fourth is the profit gained by the north in the south. Therefore, both terms represent balance of capital.

Southern capital accumulation. The capital accumulation in the south is given by

$$\dot{k}_S^* = s^* r^* k_S^* - nk_S^* \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (44)$$

⁶ We had $\frac{K}{L_N} = k_N$ in the last section because we were not considering the capital movement.

However, in both cases k_N represents the northern capital per northern worker.

Determinant relations of the model. This model is closed with 18 Equations, (2)~(7), (9), (10), (12)~(14), (18), (38), (40)~(44) and 18 endogenous variables, $f(k)$, k , w , r , L_N , L_G^* , y , k_N , y_G^* , Q , $f^*(k^*)$, k^* , r^* , p , k_S^* , λ , y^* and e^* .

The major relations are as follows. Assuming that L_N , k_S^* and k_N each take a certain value from equations (5), (18) and (44). Equation (13) then solves k^* , and based on that $f^*(k^*)$ and r^* are solved in equations (12) and (14). Using Equations (3), (4), (7) and (9) we know that both y and y_G^* are functions of k and Equation (40) shows that λ is also a function of k . Equation (38) tells us that k is a function of p , hence y , y_G^* , and λ are also functions of p . By virtue of Equations (41) and (42) both y^* and e^* are also functions of p . Taking all these into consideration, p is solved in Equation (43) and all other variables will then be determined.

3.2. Relaxation of Immigration Policy by the North

This section examines how the income in both regions is affected—in the short-run where both k_S^* and k_N are fixed and in the long run where both are flexible—when the north increases the immigrants intake from the south.

Short-run equilibrium analysis. Let us start with how the relaxed immigration policy affects the terms of trade. Equation (43) tells us that

$$\frac{dp}{d\tau} = \frac{\mu \frac{\partial \lambda}{\partial \tau} + \lambda \mu_2 \frac{\partial e^*}{\partial \tau} - r y_G^* + r^* k^* \frac{\partial \lambda}{\partial \tau} - p \tilde{m}}{\Gamma} \quad \dots \quad \dots \quad \dots \quad (45)$$

On top of that, we know that $\Gamma < 0$ by virtue of the stability of the balance of trade equilibrium equation. The first and second term of the numerator represents the changes of the south's import of northern consumption goods. The third and fourth term shows the change in capital balance and the fifth term shows the change in the north's import of southern goods.

The sign of $\frac{dp}{d\tau}$ is undetermined because Equation (40) tells us that $\frac{\partial \lambda}{\partial \tau} < 0$

while Equations (41) and (42) tells us that $\frac{\partial e^*}{\partial \tau} > 0$. If $\frac{dp}{d\tau} > 0$, by virtue of Equation (38) r rises and k falls according to Equation (4).

Hence using Equations (3), (7) and (9) we can conclude that both w and y_G^* reduces and y increases. As for y^* , using Equation (41)

$$\frac{dy^*}{d\tau} = \frac{q y_G^*}{\lambda p} + \frac{q \tau}{\lambda p} \cdot \frac{d y_G^*}{d \tau} - \frac{q y_G^* \tau}{\lambda p^2} \cdot \frac{dp}{d \tau} \quad \dots \quad \dots \quad \dots \quad \dots \quad (46)$$

Equation (40) could be rewritten as

$$\frac{d\lambda}{d\tau} = \frac{-k - (1 + \tau) \frac{dk}{d\tau}}{k^*} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (47)$$

The sign of $\frac{d\lambda}{d\tau}$ is unable to be determined because $\frac{dk}{d\tau} < 0$. Therefore, the sign of $\frac{dy^*}{d\tau}$ is also undetermined. Using the same logic, if $\frac{dp}{d\tau} < 0$ then k , w and y_G^* will increase and both y and r will fall. In this case Equation (47) says that λ will fall because k increases. Thus, it is clear that y^* will increase by virtue of Equation (46).

These results differ from that of Wooton (1985) for y always increases while y_G^* decreases in the short-run. This is so because the capital-labour ratio, k , in Wooton (1985) must fall as a result of the relaxed immigration policy. However, k in this paper will increase as a result of the capital movement from the south to the north when terms of trade worsens in the state of balance of payments equilibrium, because capital is assumed to be moving arbitrarily.

Table 3 sums up the short-run results of this paper.

Table 3

Findings in This Paper (Short-run)

	y	y_G^*	py^*	p	λ
τ	\pm	\pm	\pm	\pm	\pm

Long-run equilibrium analysis. Assume that this dynamic system satisfies the stability condition at the proximity of its equilibrium value.⁷

First let's take a look at y and y_G^* . A quick reference to its determinant relations at the equilibrium shows that k^* is solved in Equation (13), p in Equation (44), r^* in Equation (14), and r in Equation (38). Hence k is solved in Equation (4) and w is solved in Equation (3). Lastly k_N and y are solved by virtue of Equations (7) and (18).

Since these determinant relations have nothing to do with τ ,

$$\frac{dy}{d\tau} = 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (48)$$

⁷Please refer Appendix 3.

$$\frac{dy_G^*}{d\tau} = 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (49)$$

Both y and y_G^* are not affected by the immigration policy at all. The increase of immigrants will reduce the north's capital-labour ratio, k , and increase its profit rate, r , in the initial phase. However, we know from Equation (38) that it will cause a shift of capital from the south to the north and eventually k will return its initial level. Therefore, r will remain fixed because p is determined by Equation (44) and k^* is determined by Equation (13). The above results differ from Wooton (1985). We can conclude that when capital could move spontaneously between the two regions such as that assumed in this paper, the north does not benefit from perfect labour mobility in the long-run.

Next, we will see how the per capita income of the south, y^* , is affected. We can rewrite Equations (40)~(43) into

$$\frac{d\lambda}{d\tau} = \frac{p\tilde{m} + (1-s^*)r^*k + ry_G^*\mu_2 \frac{\partial e^*}{\partial \tau}}{s^*r^*k^* + \mu + \lambda\mu_2 \frac{\partial e^*}{\partial \lambda}} \quad \dots \quad \dots \quad \dots \quad (50)$$

$$\frac{\partial e^*}{\partial \lambda} = \frac{-ry_G^*\tau + (1-s^*)r^*(k^*\lambda - k_s^*)}{\lambda^2\theta} \quad \dots \quad \dots \quad \dots \quad (51)$$

$$\frac{\partial e^*}{\partial \tau} = \frac{ry_G^*s^*r^*}{\lambda n} \left(1 + \frac{k\tau}{\lambda} \right) > 0 \quad \dots \quad \dots \quad \dots \quad (52)$$

The effect on y^* is undetermined because there is no telling how λ is affected.⁸ Table 4 sums up the long-run results of this paper.

Table 4

Findings in This Paper (Long-run)

	y	y_G^*	py^*	p	λ
τ	0	0	\pm	\pm	\pm

⁸ Please refer Appendix 4.

3.3. Tax Policy

This section reviews the tax policy of both the governments.

The northern tax policy. Similar to the Wooton (1985) we will tax the wage income in the north and redistribute it to the northern workers. Therefore, Equations (28) and (29) will again be utilised. By virtue of Equations (18) and (29),

$$\frac{dk_N}{d\tau} = \frac{swt}{n - sr} > 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (53)$$

We can infer from Equation (18) that $\frac{dy}{d\tau} > 0$. This shows that the effect of the redistribution to the northern workers as a result of the increase of τ will in turn raise the north's capital accumulation. Thus, the optimal policy for the north is to perfectly free the labour movement when the taxation on northern wage income is taken into consideration. This result differs from the scenario where taxation is out of the picture.⁹

Next, we will review the tax policy when the north is implementing the optimal immigration policy. That means our concern is on the tax policy when the intake of immigrant labour supply is determined in such a way so that Equation (32) holds. We know that $\frac{dk}{dt} = 0$ because k is determined using Equations (4) and (14).

Hence by using Equations (18) and (29) we can derive

$$\frac{dk_N}{d\tau} = \frac{s\tau w}{n - sr} > 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (54)$$

$\frac{d\hat{y}_N}{dt} > 0$ by virtue of Equation (29).¹⁰ Since the profit rate in the north is determined by the south's profit rate, this result is obtained because the increase in tax have increased the capital accumulation as a result of the higher tax revenue.

Based on the above results, the optimal policy for the north under a circumstance where capital movement is free is to impose a discriminative tax policy on the immigrant workers and to perfectly free the labour movement. This finding is differ from that of Wooton (1985) because k is not affected by the northern tax policy for the north's profit rate is regulated by that of the south. Wooton (1985) says that profit rate falls while k rises as a result of an increase in t .

⁹However, when the evaluation is done when tax is first brought in ($t = 0$) the result does not differ from that when tax is not accounted for because $\frac{dk_N}{d\tau} = 0$.

¹⁰This result is independent from the evaluation timing issue.

The southern tax policy. The southern government taxes the emigrants and redistributes the tax to the local southern workers. We can derive the following based on Equation (17)

$$\frac{d\lambda}{dr} = \tau y_G^* \frac{1 - (p\tilde{m}_2 + \frac{\mu_2}{\theta})}{(1 - s^*)f_s^* k_s^* + \frac{ry_G^* \tau}{p}} \dots \dots \dots (55)$$

$$\mu - \mu_2 \frac{\lambda}{\lambda}$$

The denominator takes a positive sign by virtue of stability condition. Since the denominator is the same with that in Equation (36), this result is similar to that of Wooton (1985).

Next, suppose that north has opted for the optimal policy, in which the north implements a discriminative redistribution policy against the immigrant workers and perfectly free the labour movement. In such a circumstance, when labour movement is at equilibrium, the following equation must hold.

$$(1-r)(1-t)w = pw^* \dots \dots \dots (56)$$

Since both w and pw^* are solved by Equations (13), (38) and (44) at this state, in order for (56) to hold when q changes t inevitably have to change as well. Therefore

$$\frac{dt}{dq} = -\frac{1-t}{1-r} < 0 \dots \dots \dots (57)$$

In order to stop the labour movement caused by the increase in q , the northern government is left with only one option—that is to reduce the tax levied on labour income.

4. CONCLUSIONS

We have examined in this paper how the north's relaxation of immigration policy affects the per capita income in both regions, in a situation where capital movement is free in both regions.

The major findings are as follows.

In the short-run, the relaxation of immigration policy by the north will affect the terms of trade by way of the balance of payments equilibrium equation. However, when the terms of trade worsen, the income per northern worker reduces while the income per immigrant worker increases.

The relaxation of immigration policy in Wooton (1985) spontaneously reduces the north's capital-labour ratio, however, this paper shows that capital movement from the south to the north occurs even when the terms of trade worsen, as a result rising k .

In the long-run, the north's relaxation of immigration policy does not affect the per capita income of both the northern workers as well as the immigration workers.

This paper illustrates the following mechanism—when the immigration policy is relaxed, k initially falls but since the profit rate of north, r , will increase because of that, capital movement occurs at an instant from the south to the north, returning k to the starting level. This result significantly differs from that of Wooton (1985) in which it says the north does not have any incentive to free the labour movement in the long-run under a free capital mobility regime.

In a scenario where the north implements a discriminative redistribution policy against the immigrant workers, the per capita income in the north will increase because the redistribution of income from the immigrants to the northern local workers occurs as a result of the relaxation of the immigration policy.

The optimal policy for the north will then be to completely free labour mobility and to implement a discriminative redistribution policy against the immigrant workers.

This paper has the following limitations. In a short-run analysis, it is assumed that capital moves spontaneously. This paper does not account for the difference in quality of labour. Skilled and unskilled labour are not treated accordingly.

These are the remaining issues to be solved in future studies.

Appendices**APPENDIX 1**

The following Jacobians could be derived from Equations (18) and (19).

$$a_{11} = s \frac{\partial y}{\partial k_N} - n$$

$$a_{12} = 0$$

$$a_{21} = \frac{\partial p}{\partial k_N} s^* f'^*(k^*) \lambda$$

$$a_{22} = \frac{\partial p}{\partial \lambda} s^* f'^*(k^*) \lambda$$

Assuming $a_{11} < 0$ and $\frac{\partial p}{\partial \lambda} < 0$, hence $\text{trace} < 0$ and $|\det| > 0$ and this is stable.

APPENDIX 2

$$1 - \frac{\mu_2}{p} = 1 - \frac{\mu}{e^* p} = \frac{e^* - \mu}{e^* p}$$

By virtue of Equations (15)~(17) the numerator of Equation(58) is

$$e^* - \mu = p f^*(k^*) + \frac{q y_G^* \tau}{\lambda} + \frac{p(m + \tilde{m}) + q y_G^* \tau}{\lambda} > 0$$

Hence $1 - \frac{\mu_2}{p} > 0$.

APPENDIX 3

By virtue of Equation (18), $sy = nk_N$ at long run equilibrium. Combine this fact with (7), (38) and (44),

$$k_N = \frac{sw}{n(1 - \frac{s}{s^*})} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (60)$$

It is necessary that $s^* > s$ in order to obtain a meaningful solution for (60). As for dynamic Equations (18) and (44), the Jacobians at the proximity of the equilibrium values $(k_S^*)_e$ and $(k_N^*)_e$ are

$$b_{11} = sr^*(k_N - k) \frac{\partial p}{\partial k_N} + n\left(\frac{s}{s^*} - 1\right)$$

$$b_{12} = sr^*(k_N - k) \frac{\partial p}{\partial k_S^*}$$

$$b_{21} = s^* r^* \frac{\partial p}{\partial k_N}$$

$$b_{22} = s^* r^* \frac{\partial p}{\partial k_S^*}$$

So

$$b_{11} + b_{22} = n\left(\frac{s}{s^*} - 1\right) + sr^*(k_N - k) \frac{\partial p}{\partial k_N} + s^* r^* \frac{\partial p}{\partial k_S^*} < 0 \quad \dots \quad (61)$$

$$b_{11}b_{22} - b_{21}b_{12} = nr^*(s - s^*) \frac{\partial p}{\partial k_S^*} > 0 \quad \dots \quad \dots \quad \dots \quad (62)$$

Are the stability conditions. Both $\frac{\partial p}{\partial k_N}$ and $\frac{\partial p}{\partial k_S^*}$ could be rewritten as the following based on Equations (40) and (43)

$$\frac{\partial p}{\partial k_N} = \frac{-pm_2(1-s)r + \frac{\mu}{k^*} + \lambda\mu_2 \frac{\partial e^*}{\partial k_N}}{\Gamma} \quad \dots \quad \dots \quad \dots \quad (63)$$

$$\frac{\partial p}{\partial k_S^*} = \frac{s^* r^* + \frac{\mu}{k^*} + \lambda\mu_2 \frac{\partial e^*}{\partial k_S^*}}{\Gamma} \quad \dots \quad \dots \quad \dots \quad \dots \quad (64)$$

$\frac{\partial p}{\partial k_S^*}$ takes a minus sign by virtue of (60) and (62). However, the sign of $\frac{\partial p}{\partial k_N}$ is

undetermined because from Equations (40) and (42) $\frac{\partial e^*}{\partial k_N} < 0$.

APPENDIX 4

At long run equilibrium, by virtue of Equation (44) $p = \frac{n}{s^* r^*}$ and terms of trade, p , is a constant.

We can derive the following using Equations (40)~(43)

$$\frac{d\lambda}{d\tau} = \frac{p\tilde{m} + (1-s^*)r^*k + qy_G^* - \lambda\mu_2 \frac{\partial e^*}{\partial \tau}}{s^*pr^*k^* + \mu + \lambda\mu_2 \frac{\partial e^*}{\partial \lambda}} \quad \dots \quad \dots \quad \dots \quad (65)$$

The sign of Equation (65) could not be determined because

$$\frac{\partial e^*}{\partial \lambda} = \frac{-qy_G^* + (1-s^*)r^*(k^*\lambda - k_S^*)}{\lambda^2\theta} \quad \dots \quad \dots \quad \dots \quad \dots \quad (66)$$

$$\frac{\partial e^*}{\partial \tau} = \frac{qy_G^*s^*r^*}{\lambda n} \left(1 + \frac{k\tau}{\lambda}\right) > 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad (67)$$

We will take a look at the balance of payments equilibrium Equation (43) to see what does all this mean. Substitute Equation (40) into Equation (43),

$$\begin{aligned} & p\{m[p, (1-s)y] + \tilde{m}[p, (1-q)y_G^*]\tau\} \\ &= s^*pr^*\{(1+\tau)k + k^* - k_N\}k_S^* + \lambda\mu\left[\frac{1}{p}, e^*\right] - qy_G^*\tau + pr^*\{k_N - (1+\tau)k\} \end{aligned} \quad (68)$$

When τ increases the immigrants' demand for the southern goods to the left of the equation increases, too. On the other hand, the third and fourth term to the right—balance of capital—works favourably for the south. However, there is no telling if the south's demand for the northern goods—the second term on the right—will fall or otherwise. Hence $\frac{d\lambda}{d\tau}$ is undetermined.

Take an example where $\frac{\partial e^*}{\partial \lambda} > 0$. When $\frac{\partial e^*}{\partial \tau} > 0$ is extremely large, the immigration policy of the north magnifies the demand for northern goods. Hence it is possible that $\frac{d\lambda}{d\tau} < 0$ since the north's immigration policy exerts pressure on the south's capital import from the north.

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